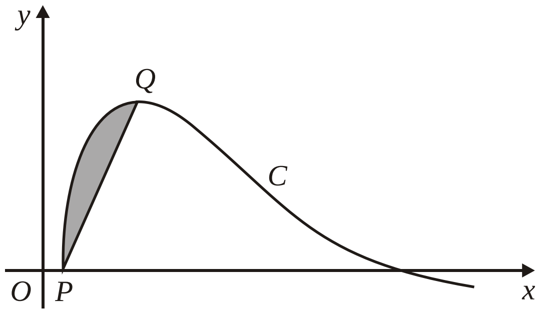
1.



The figure above shows a sketch of part of the curve C with equation

$$y = \sin(\ln x), \qquad x \geq 1.$$

The point Q, on C, is a maximum.

(a) Show that the point $P(1, 0)$ lies on C.

(1)

(b) Find the coordinates of the point Q.

(5)

(c) Find the area of the shaded region between C and the line PQ.

(9)
(Total 15 marks)

2. $$f(x) = (x^2 + 1) \ln x, \qquad x > 0.$$

(a) Use differentiation to find the value of $f'(x)$ at $x = e$, leaving your answer in terms of e.

(4)

(b) Find the exact value of $\int_1^e f(x)\,dx$

(5)
(Total 9 marks)

1. (a) $x = 1$; $y = \sin(\ln 1) = \sin 0 = 0$

$\therefore$ P = (1, 0) and P lies on C — B1 c.s.o. — 1

(b) $y' = \frac{1}{x}\cos(\ln x)$ — M1, A1

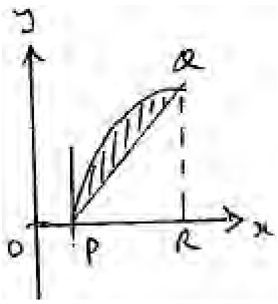
$y' = 0$ at Q $\therefore \cos(\ln x) = 0$ $\therefore \ln x = \frac{\pi}{2}$ — M1

$x = e^{\frac{\pi}{2}}$

$\therefore Q = \left(e^{\frac{\pi}{2}}, \sin(\ln e^{\frac{\pi}{2}})\right)$ — A1

$= (e^{\frac{\pi}{2}}, 1)$ — A1 — 5

(c)



Area = $\int_1^{e^{\frac{\pi}{2}}} \sin(\ln x)\mathrm{d}x$ – Area $\Delta\, PQR$ (correct approach) — M1

Area $\Delta\, PQR$ = $\frac{1}{2} \times 1 \times (e^{\frac{\pi}{2}} - 1)$ — B1

for integral; let $\ln x = u$ $\therefore x = e^u$ (substitution) — M1

$\frac{1}{x}dx = du$ $\therefore dx = e^u du$

$\underline{\text{F}} = \int_0^{\frac{\pi}{2}} \sin u.(e^u du)$ — A1

$= \left[e^u \sin u\right]_0^{\frac{\pi}{2}} - \int e^u \cos u du$ — M1

$= e^{\frac{\pi}{2}} - \left[e^u \cos u\right]_0^{\frac{\pi}{2}} - \int e^u \sin u du$ — M1

$\therefore 2\text{I} = e^{\frac{\pi}{2}} + 1$

$\text{I} = \frac{1}{2}(1 + e^{\frac{\pi}{2}}) = 1$ (I) A1

$\therefore \text{Area} = \frac{1}{2}(1 + e^{\frac{\pi}{2}}) - \frac{1}{2}(-1 + e^{\frac{\pi}{2}}) = 1$ A1

[9]

2. (a) $f'(x) = (x^2 + 1) \times \frac{1}{x} + \ln x \times 2x$ M1 A1

$f'(e) = (e + 1) \times \frac{1}{e} + 2e = 3e + \frac{1}{e}$ M1 A1 4

(b) $(\frac{x^3}{3} + x)\ln x - \int (\frac{x^3}{3} + x)\frac{1}{x}dx$ M1 A1

$= (\frac{x^3}{3} + x)\ln x - \int (\frac{x^3}{3} + 1)dx$

$= \left[(\frac{x^3}{3} + x)\ln x - (\frac{x^3}{9} + x)\right]_1^e$ A1

$= \frac{2}{9}e^3 + \frac{10}{9}$ M1 A1 5

[9]

1. This was the question in which many candidates earned their highest marks. It was also the one for which most S marks were gained. Virtually all candidates scored the first mark. Differentiation was generally good in part (b) and many candidates scored all 5 of these marks. A common error was to state that ln x=1. There were also many good attempts at part (c). Nearly all recognized the need to take the difference of two areas. Those who sought to find the area of the triangle by forming the equation of the line and then integrating usually came unstuck in a mass of algebra and they rarely obtained the correct value. Fortunately most simply used half the base x height! Integration of y was usually well done. Similar numbers of candidates used direct integration by parts ($x$sin(ln x) etc.) as used the substitution u=ln x, resulting ine^u sin u du. Many were able to complete the two cycles of parts and obtain the correct answer.

2. The product rule was well understood and many candidates correctly differentiated f(x) in part (a). However, a significant number lost marks by failing to use ln e = 1 and fully simplify their answer.

Although candidates knew that integration by parts was required for part (b), the method was not well understood with common wrong answers involving candidates mistakenly suggesting that $\int \ln x \, \mathrm{d}x = \frac{1}{x}$ and attempting to use $u = x^2 + 1$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = \ln x$ in the formula $\int u \frac{\mathrm{d}v}{\mathrm{d}x} \mathrm{d}x = uv - \int v \frac{\mathrm{d}u}{\mathrm{d}x} \mathrm{d}x$.

Candidates who correctly gave the intermediate result $\left[\left(\frac{x^3}{3} + x\right)\ln x\right]_1^{\mathrm{e}} - \int_1^{\mathrm{e}} \left(\frac{x^3}{3} + x\right)\frac{1}{x} \mathrm{d}x$ often failed to use a bracket for the second part of the expression when they integrated and went on to make a sign error by giving $-\frac{x^3}{9} + x$ rather than $-\frac{x^3}{9} - x$.